What you should learn from Recitation 8: Application of complexification

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- There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.

Complexification for nonhomog. ODE

Let $f(r) = a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0$.

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- Instead of taking the real part, now take the imaginary part of $\tilde{P}(t)$,

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  which would be a particular solution to this ODE.

Summary of the method of undetermined coefficient

So to sum up,

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- Find its complementary solutions by solving the characteristic equation $f(r) = 0$.
- By separating terms and complexification, we only need to focus the case when

  $$g(t) = e^{\alpha t} p_m(t)$$

  where $p_m(t)$ is a polynomial of degree m.

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$$P(t) = e^{\alpha t}(A_m t^m + A_{m-1} t^{m-1} + \cdots + A_1 t + A_0).$$

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  Plug $P(t)$ into the ODE, compute $f(D)P$

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Plug $P(t)$ into the ODE, compute $f(D)P$ and compare it with $g(t)$

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$$P(t) = e^{\alpha t}(A_m t^m + A_{m-1} t^{m-1} + \cdots + A_1 t + A_0).$$

  Plug $P(t)$ into the ODE, compute $f(D)P$ and compare it with $g(t)$ to determine the coefficients $A_m, A_{m-1}, \cdots, A_0$.

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  Plug $P(t)$ into the ODE, compute $f(D)P$ and compare it with $g(t)$ to determine the coefficients $A_m, A_{m-1}, \cdots, A_0$.
- If the first try fails, multiply $P(t)$ by t and try again. If second try fails, multiply $P(t)$ by t and try again, ...

Summary of the method of undetermined coefficient

So to sum up, for the nonhomogeneous ODE

$$f(D)y = a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = g(t)$$

- One can find a particular solution by trying

$$P(t) = e^{\alpha t}(A_m t^m + A_{m-1} t^{m-1} + \cdots + A_1 t + A_0).$$

  Plug $P(t)$ into the ODE, compute $f(D)P$ and compare it with $g(t)$ to determine the coefficients $A_m, A_{m-1}, \cdots, A_0$.
- If the first try fails, multiply $P(t)$ by t and try again. If second try fails, multiply $P(t)$ by t and try again, ...

Question: How many times do you have to fail?

How many times do you have to try

Theorem

If α is a root

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Theorem

If α is a root of multiplicity s,

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Example:

$$y''' - 3y'' + 3y' + y = e^t$$

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- The characteristic equation: $r^3 - 3r^2 + 3r - 1$

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- The characteristic equation: $r^3 - 3r^2 + 3r - 1 = (r-1)^3$

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- The characteristic equation: $r^3 - 3r^2 + 3r - 1 = (r-1)^3 = 0$.

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Example:

$$y''' - 3y'' + 3y' + y = e^t$$

- The characteristic equation: $r^3 - 3r^2 + 3r - 1 = (r-1)^3 = 0$. So $r = 1$ with multiplicity 3.

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- The characteristic equation: $r^3 - 3r^2 + 3r - 1 = (r-1)^3 = 0$. So $r = 1$ with multiplicity 3. So The complementary solution is

$$y(t) = C_1 e^t + C_2 te^t + C_3 t^2 e^t.$$

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- The fourth try, $P(t) = At^3 e^t$ would succeed.

Remark: If $g(t) = te^t$, you still need to try the fourth time, although you don't see why the third try fails from the above argument.

Modification of the algorithm

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Challenging Problem:

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$$P(t) = t^s$$

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$$P(t) = t^s e^{\alpha t}(A_m t^m + A_{m-1} t^{m-1} + \cdots + A_1 t + A_0).$$

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Plug $P(t)$ into the ODE, compute $f(D)P$ and compare it with $g(t)$

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Homework Problem 4.3.2

Find the general solution to the ODE

$$y^{(4)} - y = 3t + \cos t$$

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- The characteristic polynomial

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- One can easily find the particular solution for the ODE to the first term

  $$y^{(4)} - y = 3t$$

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- One can easily find the particular solution for the ODE to the first term

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  Since 0 is not a root of the char. eqn, the first try $P(t) = At + B$ would succeed.

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Find the general solution to the ODE

$$y^{(4)} - y = 3t + \cos t$$

- Compute $f(D)P$

$$f(D)P = -At - B$$

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Find the general solution to the ODE

$$y^{(4)} - y = 3t + \cos t$$

- Compute $f(D)P$

$$f(D)P = -At - B = 3t \Rightarrow A = -3, B = 0$$

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$$y^{(4)} - y = 3t + \cos t$$

- Compute $f(D)P$

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  So a particular solution is $P(t) = -3t$
- Now let's look at the ODE to the second term

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- Since i is a root of multiplicity 1,

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- Now let's look at the ODE to the second term

$$y^{(4)} - y = \cos t$$

- Complexify!

$$\tilde{y}^{(4)} - \tilde{y} = e^{it}$$

- Since i is a root of multiplicity 1, the first try will fail and one should try $\tilde{P}(t) = Ate^{it}$.

Homework Problem 4.3.2

- Simplify $f(D)\tilde{P}$ by exponential shift law:

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$$f(D)\tilde{P}$$

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- Simplify $f(D)\tilde{P}$ by exponential shift law:

$$f(D)\tilde{P} = f(D)(e^{it}At)$$

Homework Problem 4.3.2

- Simplify $f(D)\tilde{P}$ by exponential shift law:

$$\begin{aligned} f(D)\tilde{P} &= f(D)(e^{it}At) \\ &= e^{it}(f(D+i)At) \end{aligned}$$

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- Compute $f(D+i)At$: Since $f(r) = (r+1)(r-1)(r+i)(r-i)$,

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$$f(D+i)At$$

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$$f(D+i)At = (D+i+1)$$

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$$f(D+i)At = (D+i+1)(D+i-1)$$

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- Simplify $f(D)\tilde{P}$ by exponential shift law:
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$$f(D+i)At = (D+i+1)(D+i-1)(D+2i)$$

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- Simplify $f(D)\tilde{P}$ by exponential shift law:

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$$f(D+i)At = (D+i+1)(D+i-1)(D+2i)D$$

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$$f(D+i)At = (D+i+1)(D+i-1)(D+2i)DAt$$

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Book Problem 4.3.16

Find the general solution to the ODE

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$$y^{(4)} + 4y'' = \sin 2t + te^t + 4$$

- Since $2i$ is a root of multiplicity one, the template $\tilde{P}(t) = Ae^{it}$ will fail and one should try $\tilde{P}(t) = Ate^{it}$.
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- Since 1 is not a root of the char. eqn., the first try $P(t) = (At + B)e^t$ would succeed.

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So $5A = 1, 12A + 5B = 0$

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So $5A = 1, 12A + 5B = 0 \Rightarrow A = 1/5, B = -12A/5 = -12/25$

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$$P(t) = e^t(\frac{1}{5}t - \frac{12}{25})$$

Book Problem 4.3.16

Find the general solution to the ODE

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Book Problem 4.3.16

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- Compute $f(D)P$:

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- Compute $f(D)\tilde{P}$. Notice $f(r) = r^2(r+1+i)(r+1-i)$:

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The End